Practice Exam Introduction to Logic

(From January 26th, 2016)

- ☞ Only write your student number at the top of the exam, not your name. Also write your student number at the top of any additional pages.
- ☞ Use a blue or black pen (so no pencils, red pen or marker).
- ☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).
- ☞ Only hand in your definite answers. You can take any drafts home.
- ☞ When you hand in your exam, wait until the supervisors have checked whether all information is complete. They will indicate when you can leave.

Good luck!

Part A

A1: translating propositional logic Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. Alice will not be fined, unless the policeman is grumpy and tired or Alice is drunk.

b. If neither Alice nor Deepika likes the book, then I will not buy it.

A2: translating to predicate logic Translate the following sentences to *first-order logic*. Do not forget to provide the translation key. The domain of discourse is the set of all citizens of the United States of America.

a. If Trump hates everyone except himself, then everyone except Trump hates him.

b. Although no Democrat shouts louder than Sanders, Jason prefers Sanders to Clinton.

A3: formal proofs Give formal proofs of the following inferences. Do not forget the justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} (A \vee \neg B) \to C \\ \hline (A \to C) \wedge (\neg C \to B) \end{array}$$

b.
$$\begin{array}{|l} \neg B \vee B \\ \hline ((B \to A) \to B) \to B \end{array}$$

c.
$$\begin{array}{|l} \forall x \forall y \forall z((R(x,y) \wedge R(y,z)) \to \neg R(x,z)) \\ \hline \neg \exists x R(x,x) \end{array}$$

d.
$$\begin{array}{|l} a = b \\ b = c \\ \hline \exists x R(x,c) \to \exists x R(x,a) \end{array}$$

A4: truth tables Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.

a. Check with a truth table whether the conclusion is a tautological consequence of the premise for the following argument.

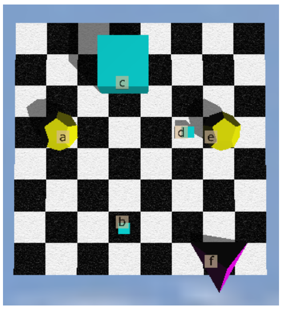
$$\begin{array}{|l} P \leftrightarrow (Q \vee R) \\ \hline (P \rightarrow Q) \vee (\neg P \rightarrow R) \end{array}$$

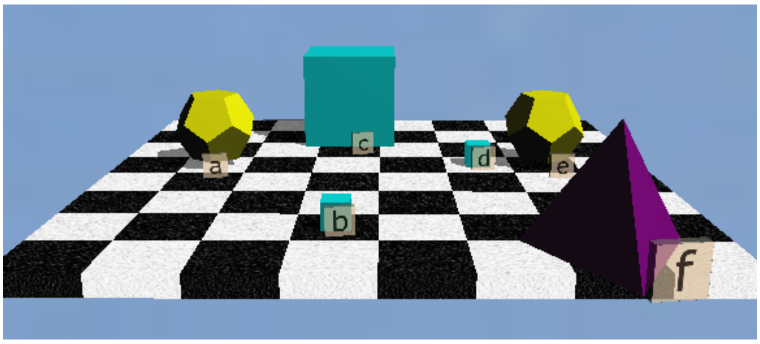
b. Check with a truth table whether the conclusion is a logical consequence of the premise for the following argument. Indicate clearly which rows are spurious.

$$\begin{array}{|l} \mathsf{Large(a)} \rightarrow \mathsf{Larger(b, a)} \\ \hline \mathsf{Larger(a, b)} \rightarrow \neg\mathsf{Large(a)} \end{array}$$

c. Check with a truth table whether the two formulas $A \leftrightarrow (B \leftrightarrow C)$ and $(A \leftrightarrow B) \leftrightarrow C$ are tautologically equivalent.

A5: Tarski's World In the world displayed below, b and d are small, c and f are large and the other objects are medium.





a. In the world displayed above, there is are at least two different cubes that are in the same column. How can you express this with one formula in the language of Tarski's World such that the formula would be true in every world with at least two different cubes that are in the same column, and false otherwise?

b. Indicate of each formula below, whether it is true or false in the world displayed above. You do not need to explain your answers.

(i) $(\mathsf{Small(c)} \vee \mathsf{SameColumn(b, c)}) \leftrightarrow \mathsf{Between(d, e, b)}$

(ii) $\mathsf{Smaller(e, f)} \rightarrow (\neg\mathsf{BackOf(d, e)} \rightarrow \mathsf{SameSize(b, d)})$

(iii) $\neg(\mathsf{LeftOf(f, d)} \leftrightarrow \mathsf{FrontOf(f, d)})$

(iv) $\forall \mathsf{x} \exists \mathsf{y}(\neg\mathsf{Small(x)} \rightarrow (\neg\mathsf{Large(x)} \rightarrow (\mathsf{BackOf(x, y)} \wedge \mathsf{Large(y)})))$

(v) $\exists \mathsf{x}(\mathsf{Large(x)} \wedge \neg\mathsf{Dodec(x)} \wedge \mathsf{RightOf(x, a)} \wedge \exists \mathsf{y}(\mathsf{x} \neq \mathsf{y} \wedge \mathsf{Large(y)}))$

(vi) $\exists \mathsf{x} \exists \mathsf{y}(\mathsf{SameRow(x, y)} \wedge \mathsf{SameSize(x, y)} \wedge (\mathsf{Cube(x)} \rightarrow \mathsf{Tet(y)}))$

(vii) $\neg\forall \mathsf{x}(\mathsf{LeftOf(x, f)} \rightarrow \mathsf{Cube(x)})$

c. Explain how the formula below can be made false by removing one object from the world displayed above.

$$\forall \mathsf{x}(\neg\mathsf{Cube(x)} \rightarrow \exists \mathsf{y}(\mathsf{BackOf(y, x)} \wedge \mathsf{Cube(y)}))$$

A6: Bonus question

Give a formal proof of the following inference. Don't forget to provide justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

$$\begin{array}{|l} \neg\forall x \exists y \neg A(x, y) \\ \hline \exists x \forall y A(x, y) \land \forall y \exists x A(x, y) \end{array}$$

Part B

B1: Normal forms propositional logic

Provide a conjunctive normal form (CNF) of the following formula. Show all of the intermediate steps.

$$(A \lor B) \leftrightarrow (\neg B \to C)$$

B2: Normal forms first-order logic

a. Provide a Prenex normal form of the following formula. Show all of the intermediate steps.
   $(A(x) \land \forall x \exists y R(x, y)) \to \exists y B(y)$

b. Provide a Skolem normal form of the following formula. Show all of the intermediate steps.
   $\exists x \forall y \exists z \forall u \exists w (R(x, y, z) \to R(x, u, w))$

c. Check the satisfiability of the Horn sentence below using the Horn algorithm. If you prefer the conditional form, you may also use the satisfiability algorithm for conditional Horn sentences.

$$A \land E \land (\neg D \lor \neg C \lor \neg B \lor A) \land (C \lor \neg A) \land (D \lor \neg E)$$

B3: Translating function symbols Translate the following sentences using the translation key provided. The domain of discourse is the set of all working people.

friend(x): x's best friend
boss(x): x's boss

a: Alan
c: Natalie
d: David

Likes(x, y): x likes y

a. The boss of Natalie's boss is David's best friend.

b. Although neither David nor Natalie is the best friend of Alan's boss, Alan's boss likes both David and Natalie.

c. If there is someone who is everyone's best friend, then that person is nobody's boss.

B4: Semantics

Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall) = \{1, 2, 3\}$ be given such that

- $\mathfrak{M}(a) = 3$
- $\mathfrak{M}(P) = \{1, 2\}$
- $\mathfrak{M}(R) = \{\langle 1, 2\rangle, \langle 2, 2\rangle, \langle 2, 3\rangle\}$

Let h be an assignment such that $h(x) = 2$, $h(y) = 3$

Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models P(a) \vee R(x, y)[h]$

b. $\mathfrak{M} \models \exists x(R(x, x) \wedge \neg P(x))[h]$

c. $\mathfrak{M} \models \forall y \exists x R(y, x)[h]$